Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
| --- | --- |
| | |

**Pearson Edexcel**
**International**
**Advanced Level**

Centre Number

Candidate Number

# Wednesday 22 May 2019

| Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA11/01** |
| --- | --- |

# Mathematics

## International Advanced Subsidiary/Advanced Level

## Pure Mathematics P1

**You must have:**
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

: explanation
$\because$ is 'because'
$\therefore$ is 'therefore'

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over*

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P 6 1 8 3 7 A 0 1 2 8

Pearson

**Answer all questions. Write your answers in the spaces provided.**

**1.** The curve $C$ has equation $y = \frac{1}{8}x^3 - \frac{24}{\sqrt{x}} + 1$

(a) Find $\frac{dy}{dx}$, giving the answer in its simplest form.

**(3)**

The point $P(4, -3)$ lies on $C$.

(b) Find the equation of the tangent to $C$ at the point $P$. Write your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(3)**

a) ① rewrite equation in easier form for differentiation

$y = \frac{1}{8}x^3 - 24x^{-1/2} + 1x^0$

$\frac{24}{\sqrt{x}} = 24x^{-1/2}$ ∵ indicies rules: $a^{\frac{b}{c}} = \sqrt[c]{a^b}$, $xa^{-b} = \frac{x}{a^b}$

$1 = 1 \times 1 = 1x^0$ ∵ indicies rule : $a^0 = 1$

② differentiate

$dy/dx = 3 \times \frac{1}{8}x^{3-1} - \left(-\frac{1}{2} \times 24x^{-\frac{1}{2}-1}\right) + 0(1x^{0-1})$

$= \frac{3}{8}x^2 - \left(-12x^{-\frac{3}{2}}\right) + 0$

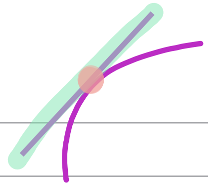
③ Simplify

$\frac{3}{8}x^2 \underbrace{- (-}_{\text{Product of 2 negatives is positive}} 12x^{-3/2})$

∴ $dy/dx = \frac{3}{8}x^2 + 12x^{-\frac{3}{2}}$

Question 1 continued

b) tangent



So shares same gradient as curve at point of intersection

∴ Substitute $x$-coordinate of $P$ into gradient function $dy/dx$ from part (a)

$P(4, -3)$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{8}(4)^2 + 12(4)^{-\frac{3}{2}} = 7.5$$

To find equation of tangent use:

line passing through $(a, b)$ and gradient $m$

equation: $(y - b) = m(x - a)$

$a = 4$
$b = -3$
$m = 7.5$

$$(y-(-3)) = 7.5(x - 4)$$

Write in the form $y = mx + c$ ← Check which form question asks for

$y + 3 = 7.5(x-4) \Rightarrow y + 3 = 7.5x - 30$

$-3 \quad y = 7.5x - 33 \quad -3$

$\therefore y = 7.5x - 33$

Q1

(Total 6 marks)

$\frac{p}{q}$ where $p, q \in \mathbb{Z}$

fraction made up of 2 integers

Leave blank

**2. Answer this question showing each stage of your working.**

(a) Simplify $\dfrac{1}{4 - 2\sqrt{2}}$

giving your answer in the form $a + b\sqrt{2}$ where $a$ and $b$ are rational numbers.

**(2)**

(b) Hence, or otherwise, solve the equation

$$4x = 2\sqrt{2}x + 20\sqrt{2}$$

giving your answer in the form $p + q\sqrt{2}$ where $p$ and $q$ are rational numbers.

**(3)**

a) rationalising Surds so that denominator becomes rational

① $\dfrac{1}{4-2\sqrt{2}}$ multiply fraction with conjugate of $4-2\sqrt{2}$

$\Rightarrow \dfrac{1}{4-2\sqrt{2}} \times \dfrac{4+2\sqrt{2}}{4+2\sqrt{2}} = \dfrac{1(4+2\sqrt{2})}{(4-2\sqrt{2})(4+2\sqrt{2})}$

② Simplify

$\dfrac{(4+2\sqrt{2})}{16+8\sqrt{2}-8\sqrt{2}-8} = \dfrac{4+2\sqrt{2}}{8} = \dfrac{\cancel{2}(2+\sqrt{2})}{\cancel{2}(4)}$

③ in the form $a+b\sqrt{2}$

$\frac{1}{4}(2+\sqrt{2}) = \frac{2}{4} + \frac{1}{4}\sqrt{2} = \frac{\cancel{2}(1)}{\cancel{2}(2)} + \frac{1}{4}\sqrt{2}$

$\therefore \frac{1}{2} + \frac{1}{4}\sqrt{2}$

$a = \frac{1}{2}$

$b = \frac{1}{4}$

Leave blank

**Question 2 continued**

b) $4x = 2\sqrt{2}x + 20\sqrt{2}$

$-2\sqrt{2}x$ $\quad$ $-2\sqrt{2}x$

$4x - 2\sqrt{2}x = 20\sqrt{2}$

$(4 - 2\sqrt{2})x = 20\sqrt{2}$

$\div(4-2\sqrt{2})$ $\quad$ $\div(4-2\sqrt{2})$

$x = \dfrac{20\sqrt{2}}{4-2\sqrt{2}}$

Another way to write $\dfrac{20\sqrt{2}}{4-2\sqrt{2}}$ is $\dfrac{1}{4-2\sqrt{2}}(20\sqrt{2})$

$\therefore \quad x = 20\sqrt{2} \times \left(\dfrac{1}{2} + \dfrac{1}{4}\sqrt{2}\right)$ → answer from part (a)

$x = 10\sqrt{2} + 10$

in the form $p + q\sqrt{2}$

$\therefore \; 10 + 10\sqrt{2}$

$p = 10$
$q = 10$

Q2

**(Total 5 marks)**

**3.**

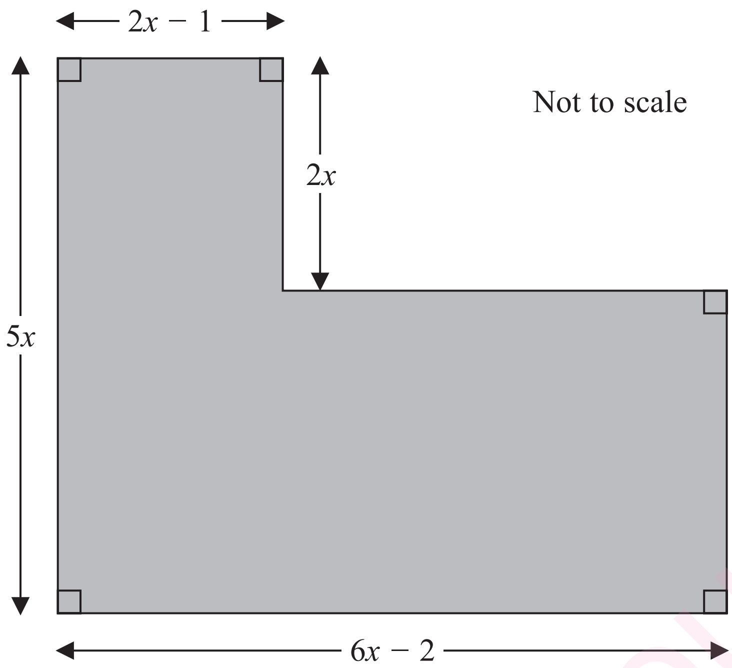



**Figure 1**

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

**(3)**

Given also that the area of the garden is less than 72 m$^2$,

(b) form and solve a quadratic inequality in $x$.

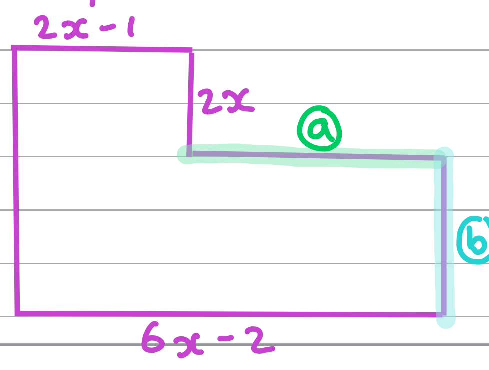
**(5)**

(c) Hence state the range of possible values of $x$.

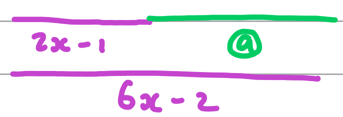
**(1)**

a) Perimeter $> 29$m

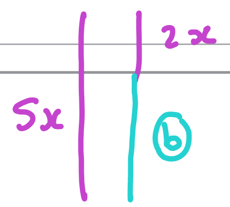
① find the perimeter.




ⓐ $= (6x-2) - (2x-1) = 4x-1$




ⓑ $= 5x - 2x = 3x$



**Question 3 continued**

$\therefore$ perimeter $= (6x-2) + 5x + (2x-1) + 2x + (4x-1) + 3x$
$= 22x - 4$

② Put perimeter into inequality

$22x - 4 > 29m$

③ rearrange to have only $x$ on one side.

$+4$: $22x - 4 > 29m$
$22x > 33m$

④ Simplify

$\div 11$: $22x > 33m$
$2x > 3m$

we want to show inequality $x > 1.5m$ $\therefore$

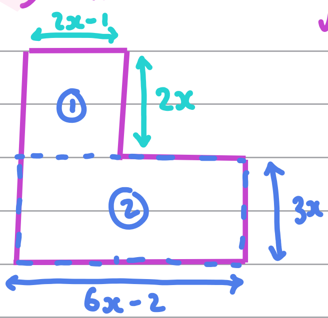
$\div 2$: $2x > 3m$
$x > \underbrace{\frac{3}{2}}_{\frac{3}{2} = 1.5} m$

$\therefore$ $x > 1.5m$

b) Area $< 72m^2$

① find area of Shape.
→ i) Split shape
ii) find area
iii) add area together.

i)




ii) $A_{①} = 2x(2x-1) = 4x^2 - 2x$
$A_{②} = 3x(6x-2) = 18x^2 - 6x$

iii) $A_{①} + A_{②} = (4x^2 - 2x) + (18x^2 - 6x)$
$= \underline{22x^2 - 8x}$ $m^2$

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Question 3 continued

② form quadratic by using inequality area $< 72$

$22x^2 - 8x < 72$

$-72 \quad 22x^2 - 8x - 72 < 0 \quad -72$

③ Simplify

$2(11x^2 - 4x - 36) < 2(0)$

$11x^2 - 4x - 36 < 0$

④ Solve quadratic

when $11x^2 - 4x - 36 = 0$

$(11x + 18)(x - 2) = 0$

you can use quadratic formula as well.

$11x + 18 = 0 \quad \therefore x_1 = -\frac{18}{11}$

$x - 2 = 0 \quad \therefore x_2 = 2$

⑤ form inequality by finding for which values of $x$, inequality is valid.

(i) $x \neq 2$ & $x \neq -\frac{18}{11}$ $\because 11x^2 - 4x - 36 < 0$

$\because$ it is not actually equal to zero

(ii) $11x^2 - 4x - 36 < 0$

you can pick any value less than or greater than 2 and test it → when $x = 3$ $\quad 11x^2 - 4x - 36 = 51$

$51 > 0 \quad \therefore x$ CANNOT be greater than 2

when $x = -\frac{19}{11}$ $\quad 11x^2 - 4x - 36 = \frac{41}{11}$

$\frac{41}{11} > 0 \quad \therefore x$ CANNOT be less than $-\frac{18}{11}$

NOT $\leq$ $\because$ not equal to

$\therefore -\frac{18}{11} < x < 2$

c) Cannot have negative number as $x$ $\because$ length / distance is always positive. in part (a) we said that $x > 1.5$

$\therefore 1.5 < x < 2$

Leave blank

**Question 3 continued**

Q3

**(Total 9 marks)**

Leave blank

**4.** Find

$$\int \frac{4x^2 + 1}{2\sqrt{x}}\,dx$$

giving the answer in its simplest form.

**(5)**

Integration method.

① write equation in an easier form to integrate

$$\frac{4x^2+1}{2\sqrt{x}} = \frac{4x^2}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = \left(\frac{4}{2} \times \frac{x^2}{\sqrt{x}}\right) + \left(\frac{1}{2} \times \frac{1}{\sqrt{x}}\right)$$

$$\frac{4}{2} \times \frac{x^2}{\sqrt{x}} = 2 \times \frac{x^2}{\sqrt{x}} = 2 \times \frac{x^2}{x^{1/2}} = 2x^{2-\frac{1}{2}} = 2x^{3/2}$$

① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

② indicies rule $\frac{a^b}{a^c} = a^{b-c}$

$$\frac{1}{2} \times \frac{1}{\sqrt{x}} = \frac{1}{2} \times \frac{1}{x^{1/2}} = \frac{1}{2}x^{-1/2}$$

① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

③ indicies rule $\frac{x}{a^b} = xa^{-b}$

$$\therefore \frac{4x^2+1}{2\sqrt{x}} = 2x^{\frac{3}{2}} + \frac{1}{2}x^{-1/2}$$

Leave blank

**Question 4 continued**

② Integration

$$\int 2x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\,dx = \left[\left(\frac{2}{\frac{3}{2}+1}x^{\frac{3}{2}+1}\right) + \left(\frac{(\frac{1}{2})}{(-\frac{1}{2}+1)}x^{-\frac{1}{2}+1}\right)\right]$$

$$= \frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + C$$

Do NOT forget $+C$ as you will lose marks for it

$$\therefore \frac{4}{5}x^{\frac{5}{2}} + x^{\frac{1}{2}} + C$$

Q4

**(Total 5 marks)**

Leave blank

**5.** (a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0$$

**(3)**

(b) Hence find all real solutions of

$$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$$

**(4)**

a) ① factorise

$x(2x^2+3x-35)=0$

$\therefore x_1 = 0$ as zero multiply any value = zero so when $x=0$ $0(2(0)^2+3(0)-35)=0$

② factorise to brackets (or can use quadratic formula)

$x(2x-7)(x+5)=0$

$2x-7=0$ $x=\frac{7}{2}$ $\therefore x_2=\frac{7}{2}$

$x+5=0$ $x=-5$ $x_3=-5$

$\therefore x = -5, 0, \frac{7}{2}$

b) ① let $x=(y-5)^2$ then substitute into equation

$(y-5)^6 = ((y-5)^2)^3 = x^3$

$(y-5)^4 = ((y-5)^2)^2 = x^2$

$\therefore$ new equation is: $2x^3+3x^2-35x=0$

② using part (a), solutions are:

$x=-5, 0, \frac{7}{2}$

**Question 5 continued**

③ Substitute $x$ with $(y-5)^2$.

1) $(y-5)^2 \neq -5$ undefined! $\because$ $y-5 = \sqrt{-5}$ ← NOT POSSIBLE to have root of negative number

2) $(y-5)^2 = 0$ → $(y-5)^2 = 0$ $\therefore$ $y_2 = 5$

3) $(y-5)^2 = \frac{7}{2}$

④ Solve for $y$ when $x = \frac{7}{2}$

$(y-5)^2 = y^2 - 10y + 25$

$y^2 - 10y + 25 = \frac{7}{2}$

$y^2 - 10y + \frac{43}{2} = 0$

Solve using quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = -10$

$c = \frac{43}{2}$

$\frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(\frac{43}{2})}}{2(1)}$

$= \frac{10 \pm \sqrt{14}}{2} = \frac{10}{2} \pm \sqrt{\frac{14}{4}} = 5 \pm \sqrt{\frac{7}{2}}$

$\therefore y = 5, \ 5 + \sqrt{\frac{7}{2}}, \ 5 - \sqrt{\frac{7}{2}}$

Q5

**(Total 7 marks)**

Leave blank

**6.** The line with equation $y = 4x + c$, where $c$ is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of $c$.

**(4)**

(b) Hence find the coordinates of the point of intersection.

**(3)**

a) ① equate the two equations to find common point (the intersection / meeting point)

$x(x-3) = 4x + c$

② form equation

expand brackets

$x(x-3) = 4x + c$

$x^2 - 3x = 4x + c$

$-4x$ $\quad x^2 - 7x = c \quad$ $-4x$

$-c$ $\quad x^2 - 7x - c = 0 \quad$ $-c$

③ Use the discriminant formula for one intersection.

$b^2 - 4ac = 0$ for one real solution

$x^2 - 7x - c = 0$

$a = 1$

$b = -7$

$c = -c$

$(-7)^2 - 4(1)(-c) = 0$

$49 - 4(-c) = 0$

negative x negative = positive

$49 + 4c = 0$

④ Solve for c

$49 + 4c = 0$

$-49$ $\quad 4c = -49 \quad$ $-49$

$\div 4$ $\quad c = -\frac{49}{4} = -12.25 \quad$ $\div 4$

$\therefore \quad c = -12.25$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question 6 continued

c) To find coordinates of intersection, equate both equations & solve for $x$ then find $y$.

Equation from part (a) is $x^2 - 7x - c = 0$

① Substitute $c$ using answer from part (a)

$c = -12.25$

$x^2 - 7x - (-12.25) = 0$

$x^2 - 7x + 12.25 = 0$ This is our equation to solve.

② Solve equation for $x$.

$x^2 - 7x + 12.25 = 0$

Solve using quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$
$b = -7$
$c = 12.25$

$\dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12.25)}}{2(1)}$

$= \dfrac{7 \pm \sqrt{0}}{2} = \dfrac{7}{2}$

$\therefore x = \dfrac{7}{2}$

③ Substitute $x$ into equation of either curve or line to find $y$

$y = x(x - 3)$

$y = \left(\frac{7}{2}\right)\left(\left(\frac{7}{2}\right) - 3\right) = \dfrac{7}{4}$ $\therefore y = \dfrac{7}{4}$

$\therefore$ the curve & line intersect only at $\left(\frac{7}{2}, \frac{7}{4}\right)$

Q6

**(Total 7 marks)**

**7.**

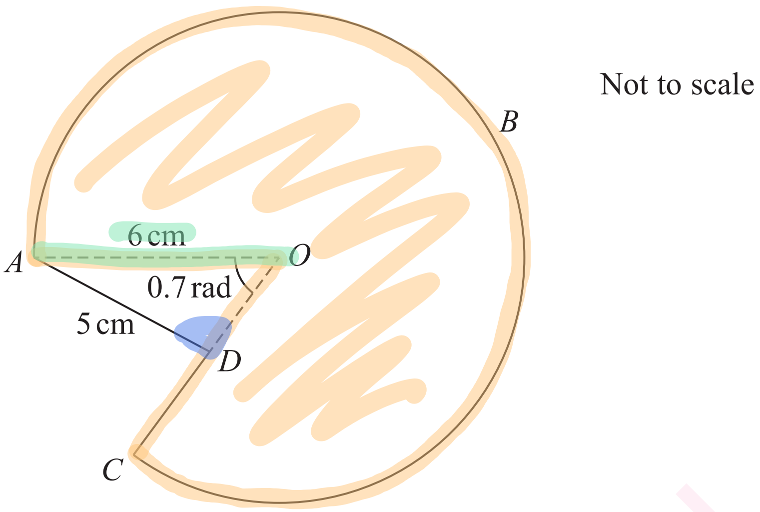


**Figure 2**

The shape $ABCDA$ consists of a sector $ABCOA$ of a circle, centre $O$, joined to a triangle $AOD$, as shown in Figure 2.

The point $D$ lies on $OC$.

The radius of the circle is 6 cm, length $AD$ is 5 cm and angle $AOD$ is 0.7 radians.

ALWAYS check units & units on calculator

(a) Find the area of the sector $ABCOA$, giving your answer to one decimal place. **(3)**

Given angle $ADO$ is obtuse,

(b) find the size of angle $ADO$, giving your answer to 3 decimal places. **(3)**

(c) Hence find the perimeter of shape $ABCDA$, giving your answer to one decimal place. **(4)**

a) Area of a Sector formula is $A = \frac{1}{2} r^2 \theta$

radius = 6cm

Total $\theta$ of a circle is $2\pi$ rad ($360°$) so the $\theta$ of this sector is : $\theta = (2\pi - 0.7)$ rad

$\therefore$ Area $= \frac{1}{2}(6)^2(2\pi - 0.7) = 18(2\pi - 0.7) = 100.49733...$

$\therefore$ Area $= 100.5 \text{ cm}^2$ (1dp)



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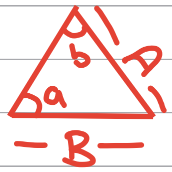
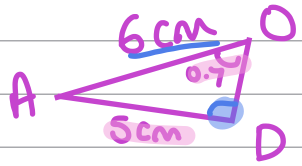
Question 7 continued

b) Use Sine rule

$$\frac{\sin a}{A} = \frac{\sin b}{B}$$




$$\frac{\sin ADO}{6} = \frac{\sin 0.7}{5}$$

x6 x6

$$\sin ADO = 6\left(\frac{\sin 0.7}{5}\right)$$

inverse sine inverse sine

$$\angle ADO = \sin^{-1}\left(6\left(\frac{\sin 0.7}{5}\right)\right)$$

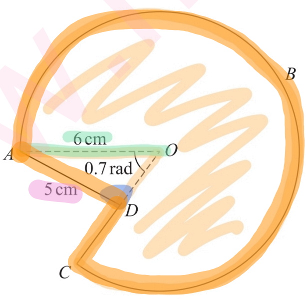
$\angle ADO = 0.8836529\ldots$ HOWEVER! this value is acute $\because$ $0.8836\ldots < \frac{\pi}{2}$ (equivalent to 90°)

This is due to sine rule sometimes producing 2 solutions

$\therefore$ Use: $\pi - 0.8836529\ldots = 2.25793\ldots$

$\therefore$ $\angle ADO = 2.258$ rad (3dp)

c)



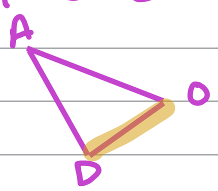

Perimeter = AD + DC + ABC
= 5 + DC + ABC

length of arc: $S = r\theta$

$S_{ABC} = 6(2\pi - 0.7)$

To find DC: CDO is radius: CDO = 6cm.

$\therefore$ DC = CDO − DO = 6 − DO




find using Sine rule.

$$\frac{\sin(\pi - 0.7 - 2.258)}{OD} = \frac{\sin 0.7}{5}$$

[Continued next page]

Question 7 continued

$OD = \left(\sin(\pi - 0.7 - 2.258)\right)\left(\frac{5}{\sin 0.7}\right)$

$= 1.416935\ldots$

$DC = 6 - 1.416935\ldots = 4.583064\ldots$

$\Rightarrow$ Perimeter $= 5 + 4.583064\ldots + 6(2\pi - 0.7)$

$= 43.082176\ldots$

$\therefore$ Perimeter $= 43.1\text{cm}$ (1dp)





P 6 1 8 3 7 A 0 1 8 2 8

**Question 7 continued**

Q7

**(Total 10 marks)**

8. The curve $C$ with equation $y = f(x)$, $x > 0$, passes through the point $P(4, 1)$.

Given that $f'(x) = 4\sqrt{x} - 2 - \dfrac{8}{3x^2}$

line perpendicular to Curve at P

normal

$90°$

C

(a) find the equation of the normal to $C$ at $P$. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

(b) Find $f(x)$.

**(5)**

a) ① find gradient of Curve at P. $f'(x)$ is gradient function so substitute $x$-value of P into $f'(x)$.

$$f'(4) = 4\sqrt{4} - 2 - \frac{8}{3(4)^2} = \frac{35}{6}$$

② find gradient of normal using formula:

$$M_{normal} \times M_{curve} = -1$$

$$M_{normal} \times \frac{35}{6} = -1$$

$\div \frac{35}{6}$ $\qquad$ $\div \frac{35}{6}$

$$M_{normal} = -\frac{6}{35}$$

③ find equation of normal using line passing through $(a, b)$ and gradient $M$

equation: $(y - b) = M(x - a)$

$a = 4$

$b = 1$

$M = -\frac{6}{35}$

$$(y - 1) = -\frac{6}{35}(x - 4)$$

Leave blank

Question 8 continued

④ rearrange into form $ax + by + c = 0$

$\times 35 \quad y - 1 = -\frac{6}{35}(x-4) \quad \times 35$

$35y - 35 = -6(x-4)$

expand brackets

$35y - 35 = -6x + 24$

$+6x \qquad +6x$

$6x + 35y - 35 = 24$

$-24 \quad 6x + 35y - 59 = 0 \quad -24$

$\therefore 6x + 35y - 59 = 0$

$a = 6$

$b = 35$

$c = -59$

b) $\int f'(x)\,dx = f(x)$

$f'(x)$ is the differential of $f(x)$.
The inverse of differentiation is integration $\therefore$ we are integrating.

① rewrite $f'(x) = 4\sqrt{x} - 2 - \frac{8}{3x^2}$

into easier form for integration.

$4\sqrt{x} = 4x^{\frac{1}{2}}$ ①

① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$-\frac{8}{3x^2} = -\frac{8}{3} \times \frac{1}{x^2} = -\frac{8}{3}x^{-2}$ ②

② indicies rule $\frac{1}{a^b} = 1a^{-b}$



P 6 1 8 3 7 A 0 2 1 2 8

Turn over

Question 8 continued

$\therefore \quad f'(x) = 4x^{1/2} - 2 - \frac{8}{3}x^{-2}$

② Integrate

$\int f'(x)\,dx = \int 4x^{1/2} - 2x^0 - \frac{8}{3}x^{-2}\,dx =$

$\left[\left(\frac{4}{(\frac{1}{2}+1)}x^{1/2+1}\right) - \left(\frac{2}{0+1}x^{0+1}\right) - \left(\frac{\frac{8}{3}}{-2+1}x^{-2+1}\right)\right] = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3}x^{-1} + C$

③ find value of $C$

Substitute $P(4, 1)$ into $f(x)$, where $f(x) = 1$ and $x = 4$.

$f(4) = \frac{8}{3}(4)^{\frac{3}{2}} - 2(4) + \frac{8}{3}(4)^{-1} + C = 1$

$-14 \quad 14 + C = 1 \quad -14$

$C = -13$

$\therefore \quad f(x) = \frac{8}{3}x^{\frac{3}{2}} - 2x + \frac{8}{3}x^{-1} - 13$

Leave blank

**Question 8 continued**

Q8

**(Total 9 marks)**

**9.**

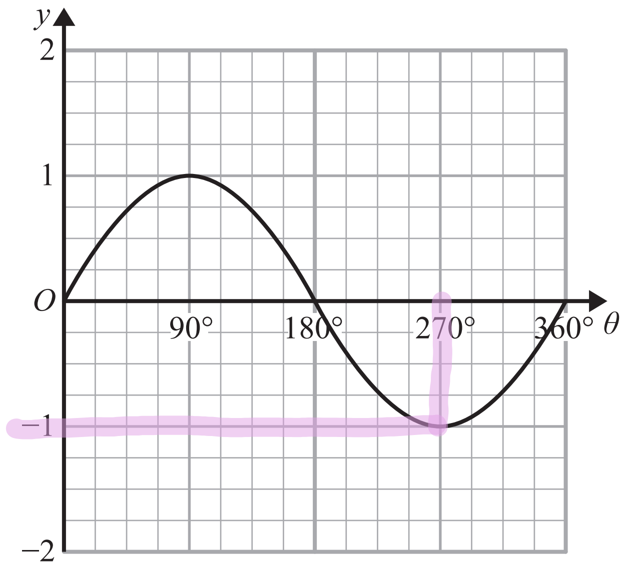



**Figure 3**

Figure 3 shows a plot of the curve with equation $y = \sin\theta$, $0 \leqslant \theta \leqslant 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leqslant \theta \leqslant 360°$$

**(2)**

A copy of Figure 3, called Diagram 1, is shown on the next page.

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta$, $0 \leqslant \theta \leqslant 360°$

(ii) $y = \tan\theta$, $0 \leqslant \theta \leqslant 360°$

**(2)**

(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 2160°$

(ii) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 1980°$

**(3)**

a) Minimum of $y = \sin\theta$ is $-1$ $-1$

$y = 4\sin\theta$ is same as $y = 4f(x)$. So vertical stretch by 4.

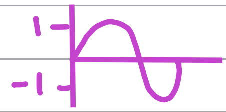
$4 \times -1 = -4$

x-axis (from figure 3) is $270°$.

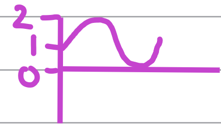
$\therefore$ Minimum $(270°, -4)$

**Question 9 continued**

b) i) $y = 1 + \sin\theta$ Same as $1 + f(x)$. Move up $\sin\theta$ by 1 unit.

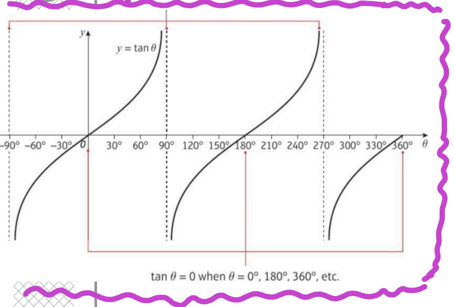


to



ii) $\tan\theta$

from textbook:



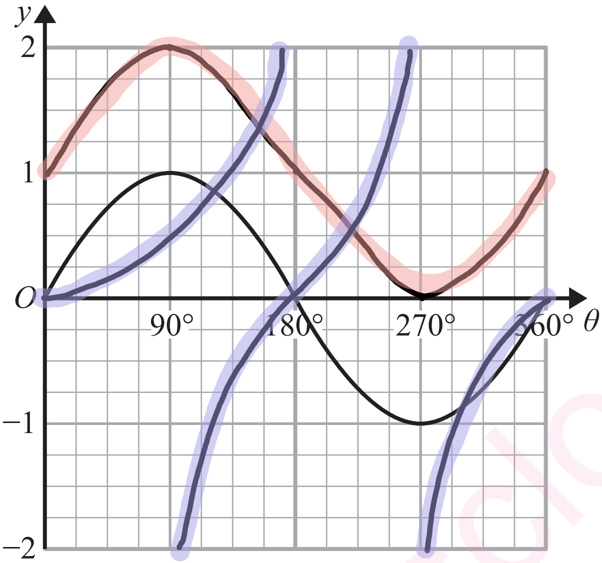
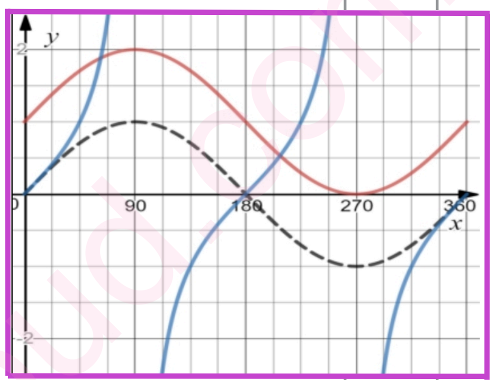




**Diagram 1**

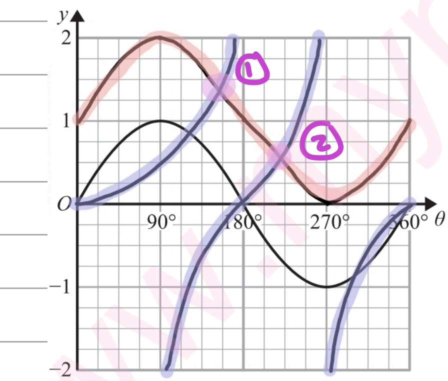
Mark Scheme:




c) i) $0 \le \theta \le 2160$ for $\tan\theta = 1 + \sin\theta$

Solutions is number of intersections.

in $0 \le \theta \le 360$ there are 2 intersections.



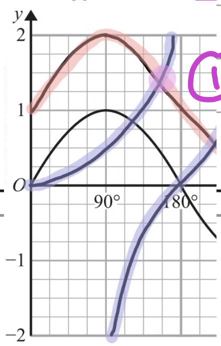
$360 \xrightarrow{\times 6} 2160$

$2 \xrightarrow{\times 6} 12$

$\therefore$ in $0 \le \theta \le 2160$ there are 12 solutions

ii) $0 \le \theta \le 1980$ for $\tan\theta = 1 + \sin\theta$

in $0 \le \theta \le 180$ there is 1 solution.



$180 \xrightarrow{\times 11} 1980$

$1 \xrightarrow{\times 11} 11$

$\therefore$ in $0 \le \theta \le 1980$ there are 11 solutions

**Q9**

**(Total 7 marks)**

Leave blank

**10.** A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the $x$-axis at the point $P$ and crosses the $x$-axis at the point $Q$.

(a) State the coordinates of the point $P$.
**(1)**

(b) Find $f'(x)$.
**(4)**

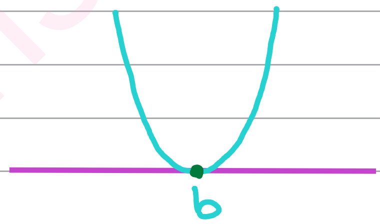
(c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where $k$ is a constant to be found.
**(3)**

The curve with equation $y = f(x + a)$, where $a$ is a constant, passes through the origin $O$.

(d) State the possible values of $a$.
**(2)**

a) Curve will 'touch' the $x$-axis when it is a quadratic.

e.g. $y = (x-b)^2$




$\therefore$ in the given equation $f(x) = (x-4)(2x+1)^2$

we do $(2x+1)^2 = 0$

$2x + 1 = 0$

$\therefore x = -\frac{1}{2}$

and as curve is touching $x$-axis, $y$ coordinate is zero

$\therefore P(-\frac{1}{2}, 0)$

b) Differentiation $\because$ $f'(x)$ is differential of $f(x)$

① expand brackets

$f(x) = (x-4)(2x+1)^2$





P 6 1 8 3 7 A 0 2 6 2 8

**Question 10 continued**

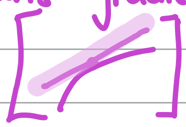
$f(x) = (x-4)(4x^2+4x+1)$
$= 4x^3 - 12x^2 - 15x - 4$

(2) differentiate

$f'(x) = 3(4x^{3-1}) - 2(12x^{2-1}) - 1(15x^{1-1}) - 0(4)$

$\therefore f'(x) = 12x^2 - 24x - 15$

c) tangent means gradient of tangent is same as gradient of curve



(1) to find gradient of tangent, substitute $x$-value into $f'(x)$ from part (b) (the gradient function)

$f'(\frac{5}{2}) = 12(\frac{5}{2})^2 - 24(\frac{5}{2}) - 15 = 75 - 60 - 15 = 0$

(2) find y-coordinate when $x = \frac{5}{2}$

$f(\frac{5}{2}) = (\frac{5}{2} - 4)(2(\frac{5}{2}) + 1)^2 = (-\frac{3}{2})(6)^2$
$= -54$

when $x = \frac{5}{2}$ $\quad y = -54$

(3) equation of tangent in form $y = k$

$(a, b)$
$y - b = m(x - a)$

as gradient $m = 0$, $\quad y - b = 0$

$y - (-54) = 0$

$\therefore y = -54$

$k = -54$

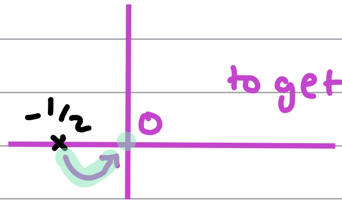

P 6 1 8 3 7 A 0 2 7 2 8

**Question 10 continued**

d) transformation.

$y = f(x+a)$ passes through origin. As $a$ is inside the brackets, this means moving left or right on the $x$-axis $a$ units.

Curve touches $x$-axis at $-\frac{1}{2}$

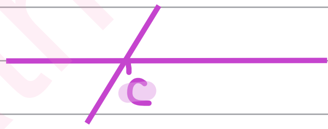



to get from $-\frac{1}{2}$ to origin, move to the right $\frac{1}{2}$ units.

This is $x + \frac{1}{2}$, but in function $f(x+a)$ is moving $x - a$ $\therefore a_1 = -\frac{1}{2}$

Curve <u>ALSO</u> passes through P at $x$-axis.

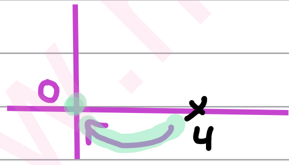
When $y = (x + c)$




line passes through $-c$.

hence for $f(x) = (x-4)$

$x - 4 = 0 \therefore x = 4$ line passes at 4.




left 4 units which is $x-4$.
$\therefore f(x+4)$ so $a = +4$

$\therefore a = -\frac{1}{2}$ and $a = 4$

Q10

**(Total 10 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

**END**